Martin’s Menace

Stewart Coffin Design #217

Martin Gardner worked for a week in vain to solve this puzzle & proclaimed it “the finest dissection puzzle of all time. It looks easy but is fiendishly difficult”.

Notes by David Beckwith, February 2024

Dave Regis gave me this puzzle as a present some time ago – many thanks Dave! The aim is to fit the 4 pentomino pieces into the tray. For the lid to fit snugly on the box, it had to be already solved, but I resisted the temptation to see the solution and tipped out the 4 pieces blindfold. I gave the puzzle quite a bit of effort for a week or so, without success. Whether I spent more time on it than Martin Gardner – who knows?!

Even though I failed, I did have some thoughts which might help to narrow down the line of attack, rather than just pushing pieces around at random. This article talks about those ideas and a computer program that I wrote to help, before putting the puzzle on a shelf to gather dust . . .

I showed Martin’s Menace to a visiting friend recently, and that prompted me to have another go. To my delight, I succeeded this time!! Spoiler alert . . . . . the solution is given later in this article.

Initial Thoughts

I have solved puzzles with pentomino pieces before, but they always cause me frustration! For example, fitting all 12 different pentominoes into a rectangle of size 3x20 or 4x15 or 5x12 or 6x10. So often I arrange 11 pieces together, only to find that the remaining pentomino space is not the shape of my remaining 12^{th} piece! However, with a little determination, solutions are not too hard to find.
Here is a link to many puzzles of this kind - [Pentomino configurations and solutions (isomerdesign.com)]

Martin’s Menace uses only 4 of the pentominoes giving an area of 4 x 5 = 20 squares. So, perhaps they might fit together in a 4x5 rectangle? If this were possible then, with only 4 pieces, surely it would be easy? But it comes with a warning of its difficulty, and after a short play it is clearly **not possible to fit in a 4x5 rectangle**.

Ah, but the tray is bigger than 4 x 5 squares – it is approximately 4.93 x 5.82 squares, giving an area of about 28.7 squares, with an area of 8.7 squares uncovered in a solution. But this doesn’t really help and leads to frustration, as in the following picture.

A single square doesn't fit at the bottom centre, overlapping the tray by just 7% of a square!

Trying to keep the sides of all 4 pieces parallel to the tray edges is doomed to failure. They must overflow a 4x5 rectangle, so they cannot fit into a tray which is smaller than 5x6 in both directions. So, **at least some of the pieces must be tilted at an angle to the edges of the tray**.

At first sight, this seems to waste space in the tray, but it also opens up intriguing extra possibilities for our awkward 4 pieces. So, I had a play . . . .

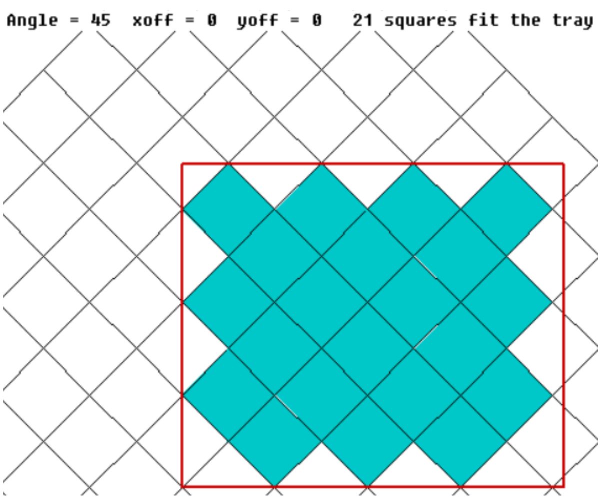
Some considerable time later, I had still failed. However, as is the case with all puzzles and mathematical problems, playing around gives some insight and feel for the task.

We need to waste as little space in the tray as possible. So, it seems sensible to fit 3 pieces snugly together, without holes in the middle. Then play with putting the configuration askew in the tray and hoping the 4th piece will fit round it. Good idea, but no solution found.

Computer Assistance

I know that there are programs to solve all pentomino puzzles where the edges of the pentominoes are parallel to the sides of the tray. But what about puzzles where the pieces are askew??

I decided to write a computer program to superimpose a square grid over the tray that the pieces had to fit into and count the number of squares that lay inside the tray. The program rotated the grid by one degree at a time and shifted both in the x and y directions. If I could find a configuration bigger than 20 squares that fitted inside the grid then that would give me more latitude with fitting pieces totalling 20 squares. The program came up with 2 solutions with area 21 squares, but they only differed by a reflection. There were no larger configurations that fitted.



Feeling hopeful, I tried to fit the 4 pieces into this shape where I could miss out one of the squares . . . but soon became convinced that it was not possible. Grump! Together with my original trials and writing the program I had run out of impetus to solve the puzzle and gave up for a while . . .

On **my second attempt** at the puzzle, I thought that perhaps the pieces might be askew to each other, as well as the sides of the tray. This would lead to apparent wasted areas, hopefully thin, but might be needed. I tried fitting a couple of pieces in a corner with the other couple askew – without success. Maybe none of the pieces snuggle neatly together? That seems counter intuitive, but maybe that's why the puzzle is so hard?

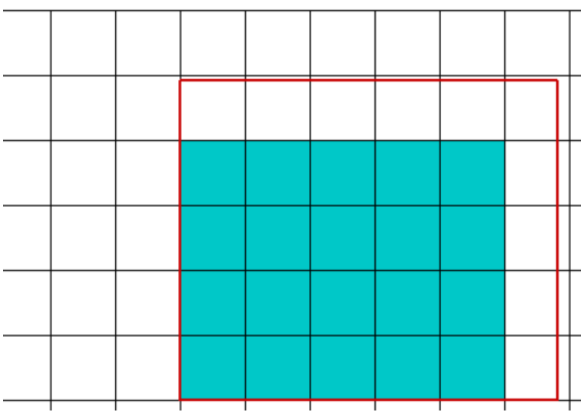
In playing with this idea, **I stumbled across the solution**!!! Whilst being delighted that I had succeeded, I felt deflated at the same time, as it felt like pure chance rather than clever logic, though one can't discount the experience gained by hours of jiggling the pieces. Regardless, I had solved it! Here is my solution: -

The following day, having rethought about the process I had followed, I wondered why my computer program had not helped me? I realised that I had only set the program to search for configurations >20 squares in size, but my solution only used a configuration of area 20!

So, I reset the program to find all possible configurations with exactly area 20 squares that fitted inside the tray. It found lots and lots because, for example, looking at the 21 square example above, it could have been translated 1,2,3 pixels to the right and still fitted. I adapted the program to allow for this, and only count one such example. There were still duplicate solutions because of rotations and reflections, but I decided to prune these manually.

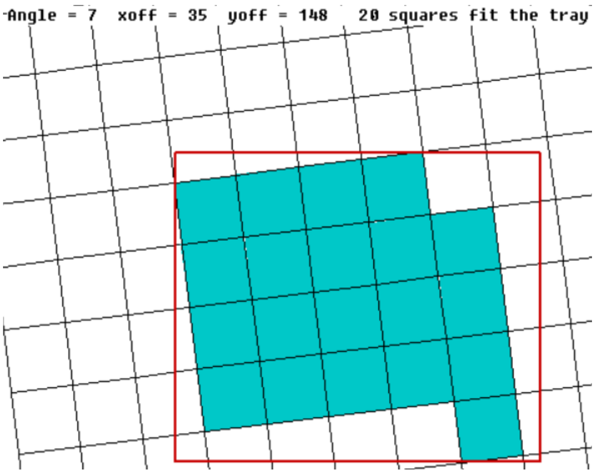
The computer found 5 different configurations with area 20 that fitted inside the grid – see below.

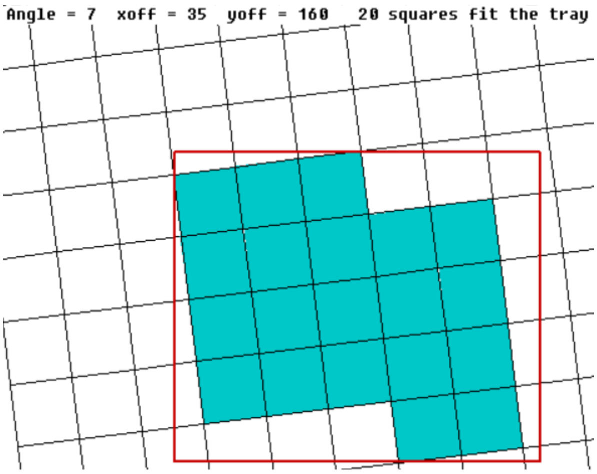
Angle = 0   xoff = 0   yoff = 0   20 squares fit the tray

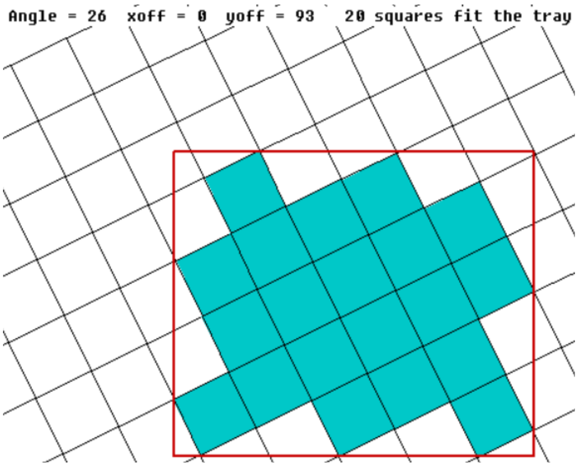


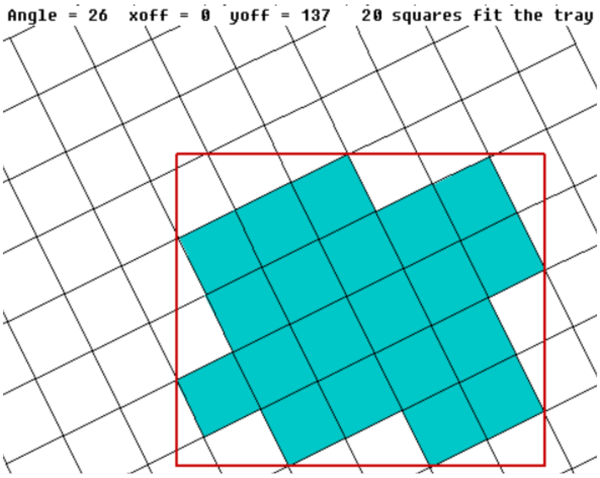
This is the trivial case that has already been dismissed.

Here are the other 4: -









I printed paper templates of these shapes of the correct size and tried fitting the 4 pentominoes. This was like the more traditional pentomino problems I had done before. It did not take long to convince myself that the solution I had already found was the only solution. If only I had used this approach before!

Of course, I have not proven that the solution is unique! I did not systematically and exhaustively check each of the 6 configurations above for solutions. It would be relatively straightforward to program this, but I did not think that it was worth the effort.

Secondly, perhaps my program missed some possible 20 square configurations? My measurements of the tray and the squares might not have been accurate enough? Also, when my program was shifting the rotated grid in the x and y directions, it produced a different number of 20 square configurations when the incremental jumps of x & y were 17 to when they were 10. The granularity of the shifts mattered. To counter this, I reduced the increments to just 1, but the program took so much longer to run. I didn't time it properly, but it was well over 12 hours. In addition, the rotations varied by just 1 degree. Perhaps more configurations would have been found had I used 0.1 degree increments? No matter how fine I made the granularity, I do not see that this approach could ever **prove** that there were only 6 possible configurations. However, I am 90+% confident in this result. Please let me know if I am wrong!

The Inventor of Martin’s Menace – Stuart Coffin

Before I was given Martin’s Menace, I already owned a copy of Stuart’s book – The Puzzling World of Polyhedral Dissections. Whilst having and enjoyed in my collection several of the easier puzzles shown in his book, my mind boggled at the complications of the 3D puzzles discussed. Way beyond me!

Searching for Stuart online, here is his Wikipedia entry Stewart Coffin - Wikipedia This gives details of his extraordinary life as a prolific puzzle maker and designer.

The following You Tube video shows his presentation at a Gathering 4 Martin in honour of Martin Gardner Stewart Coffin - Martin's Menace - G4G13 April 2018 (youtube.com)

This link AP-ART, A Compendium of Geometric Puzzles (stewartcoffin.com) goes to a compendium of Stuart’s puzzles, with his comments on each. Below is the appropriate excerpt . . .

217. Martin’s Menace or **Four Fit.** For a while I became absorbed in the form of mathematical amusements that I call square root type puzzles. In 2001 I disseminated a 20-page report, *Square Root Type Packing Problems,* with limited distribution. A condensed version was included in the 2014 Appendix. I also wrote a couple articles on the subject and contributed to a third. Out of all that came a deluge of puzzle designs. Rather than clutter up this *Compendium* with all of them, I have selected just a few of the more unusual. I consider *Martin’s Menace* the best of all my numerous designs in this category, especially because of its deceptive simplicity. It was an IPP exchange puzzle under the original name *Four Fit.* It is all based on psychology. None of the four pieces rests comfortably in a corner or even touches two sides, so where does one start? Many puzzle experts have been baffled by it, even the great Martin Gardner, hence the change of name. To quote from one of his three furtive letters concerning it: “It’s the finest dissection puzzle of all time. It looks easy but is fiendishly difficult. I wasted a week trying vainly to solve it.”

And here is his humorous finale . . .

Parting Shot

I had the strangest dream the other night. As I gingerly approached The Gates, I found myself confronted by Saint Peter with his dreaded entrance exam in hand.

"Well my son," he asked, "what have you to show for the life you have led?"

And I replied: "Well, I suppose I did have a hand, so to speak, in bringing three wonderful daughters into the world."

"Yes, we know all about that. Anything else?"

"Not a helluva a lot. Oh well, I do like to think of myself as the creator of AP-ART."

"Sure, we know all about that too. But of what significance might that be in terms of overall human destiny?"

"Ah yes, I've often wondered about that myself. I truly gave it my best effort. I suppose only time will tell."

"Good answer. And I see you've brought some of your creations along with you. Might we have a look?"

He takes a look.

"I wonder if we might have a simple one to play with here at The Gates when times get slack. How about that one? It certainly appears to be the easiest of the lot."

So I handed *Martin's Menace* to Pete (disassembled of course), continued on my way, and vanished into oblivion up amongst the clouds.